

Pricing Development Assets in Non-Sovereign Guaranteed Operations¹

by

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Summary

The article proposes a model of development agent (DA) behavior. While economic agents (EAs) maximize utility as a function of consumption, DAs maximize development impact as a function of donations. Because non-sovereign guaranteed operations (NSGOs) must be additional and non-distortionary, DAs and EAs invest on market terms. Although they may finance projects together, DAs and EAs take different risks and have different return expectations.³ As a consequence, commercial and development markets are segmented. The model is consistent with the use of third-party concessional resources in Blended Finance by Development Finance Institutions (DFIs)⁴ and Multilateral Development Banks (MDBs)⁵, and it is consistent with regular fund replenishments and general capital increases (GCIs). A relationship similar to that of the Capital Asset Pricing Model (CAPM) is derived for the expected returns of development assets in NSGOs.

A Development Impact Model

A DA maximizes development impact, D . D is a function of donations, d , and, like a utility function, is assumed to be strictly concave: $D(d)$, $D'(d) > 0$, and $D''(d) < 0$.⁶ At $t = 0$, endowed with capital $c_0 > 0$, the agent donates d_0 and transfers any remaining capital to $t = 1$ using a development investment with $t = 0$ price p_0 and $t = 1$ random payoff $\tilde{X}$. In effect, the traditional single-period consumption-investment model of an EA is restated as a single-period donation-investment model of a DA.

If the DA invests into H units of the development investment,

$$d_o = c_0 - Hp_0$$

and

$$\tilde{d} = \tilde{c} + H\tilde{X}$$

where $\tilde{d}$ is the random donation at $t = 1$ and $\tilde{c}$ is the random capital endowment at $t = 1$. Substituting d_0 and $\tilde{d}$ into

$$\max_H D(d_0) + \gamma E[D(\tilde{d})]$$

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³ Basically, DAs and EAs invest in different securities.

⁴ A DFI is defined as a sovereign-controlled bilateral that engages only in NSGOs.

⁵ An MDB is defined as a treaty-based and sovereign-owned multilateral that engages either in sovereign guaranteed operations (SGOs) or in SGOs and NSGOs.

⁶ This article glosses over technical details. For instance, donations should be non-negative. One way to achieve this is to assume that D is such that $D'(d)$ goes to ∞ as d goes to 0 from the right. See Chapter 5 in Pliska, S. (1997). *Introduction to Mathematical Finance: Discrete Time Models* (2nd ed.). Malden: Blackwell.

where $0 < \gamma \leq 1$ is the discount factor, the first-order optimality condition, which is necessary and sufficient, is

$$p_0 D'(d_0) = \gamma E[D'(\tilde{d})\tilde{X}]$$

The DA takes a position H in the development investment such that the loss in development impact when another unit of the development investment is purchased equals the gain in discounted expected development impact from the development payoff $\tilde{X}$.

Capital Endowments, Donations, and Development Impact

If the DA is thought of as a DFI or an MDB, then the capital endowments can be thought of as being provided by member states.⁷ These exogenous variables may be dependent on the DA's performance or on development needs.⁸ If the DA cannot invest, such as most United Nations (UN) agencies, then $d_0 = c_0$ and $\tilde{d} = \tilde{c}$.⁹

Most UN agencies cannot invest because they are focused on humanitarian, rather than on development, impact.¹⁰ However, the difference between humanitarian impact and development impact is not relevant to this development impact model: all DAs generate development impact solely through donations. Just as utility is derived from consumption, development impact is derived from donations. Investments—commercial and development—are means of intertemporal transfer, and their soundness hinges on their payoffs.

Consider the following three situations:¹¹

	c_0	p_0	H	d_0	c	X	d
A	10	3	1	7	11	4	15
B	10	3	0	10	11	4	11
C	10	3	1	7	11	2	13

In all three situations, c_0 , p_0 , and c are the same. **A** is better than **C** because X is greater. **B** is better than **C** because the d_0 difference is 3, while the d difference is 2.¹² While it is impossible to tell which of **A** and **B** is better, it is evident that investing is not always the ideal strategy. In particular, engaging in investments that are expected to result in losses is not an optimal policy because the possibility of deriving development impact from future donations diminishes.¹³

⁷ Member states provide financial resources in various ways, such as assessed contributions, voluntary contributions, fund replenishments, trust funds, and GCIs.

⁸ For instance, by design, MDBs are supposed to be countercyclical institutions.

⁹ Of course, imposing the constraint $H = 0$, *ceteris paribus*, cannot possibly lead to a superior optimum. If $\tilde{X} > 0$, then $p_0 > 0$.

¹⁰ The UN was not founded to maximize development impact: its primary mission was to maintain international peace and security. See <https://www.un.org/en/our-work/maintain-international-peace-and-security>.

¹¹ c , X , and d are the realizations of $\tilde{c}$, $\tilde{X}$, and $\tilde{d}$, respectively.

¹² D is strictly concave, and $0 < \gamma \leq 1$.

¹³ While it is obvious that investments that are engineered to destroy value are irrational and market-distortionary, there are organizations that prefer losing investments to donations because they believe in taking firms through an

Unless donations are recognized, the difference between a DA and an EA is unclear.¹⁴ Development institutions, such as DFIs and MDBs, are not profit-maximizing enterprises.¹⁵ They are sovereign policy tools. Their SGOs are concessional, and their NSGOs must be additional and non-distortionary. They offer grants and technical assistance, and their activities of establishing country programs and resolving market failures are not usually remunerated.¹⁶ Moreover, it is reasonable to assume that any MDB non-grant component will be eventually donated. After all, the concept of government aid is inseparable from the element of concessionality: the Development Assistance Committee (DAC) calculates Official Development Assistance (ODA) by considering the “grant equivalent” amounts of loans extended by donor countries.¹⁷

Thus, a DA maximizes development impact through an optimal donation path and executes intertemporal transfers through development investments, while an EA maximizes utility through an optimal consumption path and executes intertemporal transfers through commercial investments. Both DAs and EAs contribute to economic growth. In the case of NSGOs, they can invest in the same projects.

Blended Finance, Catalyzation, and Mobilization in NSGOs

DAs and EAs can invest in a specific project in a variety of ways. One way is Blended Finance. Blended Finance is a structuring technique that utilizes concessional resources to crowd commercial investments into projects with significant development impact.¹⁸ According to this development impact model, there cannot be direct concessionality in development investments. Since DFIs and MDBs rely on third-party concessional resources to engage in Blended Finance, this is not an issue. Direct contributions to development impact can be made through complimentary project origination, structuring, monitoring, or evaluation. In addition, DAs may take risks that EAs may not take.¹⁹

investment continuum of grants, concessional loans, and market-rate loans. The following is a standard flawed argument: given a grant of 10, it is better to

- a) deploy 10 as a money-losing loan, get back 5, deploy 5 as a money-losing loan, and get back 0 than to
- b) deploy 10 as a donation immediately.

In situation C, such organizations might claim to have delivered 23: donated 7 and invested 3 at $t = 0$, and donated 13 at $t = 1$. Since the total endowment is 21 and since the investment loses 1 (the payoff of 2 at $t = 1$ costs 3 at $t = 0$), such a claim would be false.

¹⁴ According to Finance theory, EAs take all projects with positive net present value (NPV).

¹⁵ MDBs do not pay dividends to their shareholders. Some DFIs pay dividends to their governments.

¹⁶ DFIs and MDBs do much more than make development investments. This is why, although they must not crowd out EAs in their NSGOs, DFIs and MDBs can still create development impact through donations.

¹⁷ DAC’s approach to ODA is explained here: <https://www.oecd.org/en/topics/sub-issues/oda-standards/modernising-official-development-assistance-oda.html>.

¹⁸ For two slightly different definitions, see <https://www.convergence.finance/blended-finance> and <https://www.ifc.org/en/what-we-do/sector-expertise/blended-finance/how-blended-finance-works#dfiworkinggroup>.

¹⁹ Here, the pricing of the risk is not important: the ability to take the risk is important. A project may be fairly priced and have a positive NPV, but EAs may not be allowed to take the inherent risk in their commercial investments.

Catalyzation is the commercial investment that follows in the wake of a development intervention.²⁰ For example, DFI and MDB financial instruments that introduce best-in-class governance standards or warranted local-currency lending are catalytic in nature. So are advisory services that create an enabling environment for EAs, such as policy reforms and market infrastructure. Again, only free-of-charge services and third-party concessional resources—not concessional development investments—can generate development impact.

Mobilization is also known as Cofinancing.²¹ If made *pari passu*, development investments cannot be concessional because commercial investments are on market terms, but such development investments cannot be additional. As above, gratuitous services—to participating EAs and local government authorities—can produce the desired development impact.

Essentially, to be additional to the activities of EAs, DAs need to have superior access to third-party concessional resources, be capable of managing tail risks, and offer first-rate advisory services.

Pricing Development Assets in NSGOs

If DAs must make development investments that are additional and non-distortionary, then they must invest in assets in which EAs do not invest and do so on the basis of market—or fair—prices. Specifically, if DAs and EAs finance a project together, they must take different risks and expect different returns.²² In other words, by construct, there is segmentation between commercial and development markets.

Suppose that there are N assets ($j = 1, \dots, N$) in which development investments can be made by M DAs ($i = 1, \dots, M$) with homogenous expectations. Let DA i invest a_{ij} dollars in asset j , and let asset j have a random return of $\tilde{r}_j$. Assume that $\tilde{c}_i$ depends on d_{i0} and a deterministic $r \geq 0$: $c_i = d_{i0}(1 + r)$.²³ Then,

$$c_i = d_{i0}(1 + r) = (c_{i0} - \sum_{j=1}^N a_{ij}) (1 + r)$$

and

$$\begin{aligned} \tilde{d}_i &= c_i + \sum_{j=1}^N a_{ij}(1 + \tilde{r}_j) \\ &= (c_{i0} - \sum_{j=1}^N a_{ij}) (1 + r) + \sum_{j=1}^N a_{ij}(1 + \tilde{r}_j) \\ &= c_{i0}(1 + r) + \sum_{j=1}^N a_{ij}(\tilde{r}_j - r) \end{aligned}$$

²⁰ See https://ppp.worldbank.org/public-private-partnership/sites/ppp.worldbank.org/files/2022-04/Principles_on_Crowding_in_Private_Sector_Finance_April_20.pdf

²¹ Ibid.

²² This means that *pari passu* cofinancing is not a feasible option.

²³ Having the capital endowment at $t = 1$ depend on the donation at $t = 0$ can be thought of as an incentive mechanism, and r can be thought of as an incentive coefficient. c_i can be viewed as a GCI.

This formulation of $\tilde{d}_i$ is identical to the end-of-period wealth of an EA, i , who solves the standard single-period portfolio choice problem: starting with wealth c_{i0} , the EA invest a_{ij} ($j = 1, \dots, N$) dollars in risky asset j and puts the balance in the risk-free asset.²⁴ Hence, with total initial development endowment

$$DE_0 = \sum_{i=1}^M c_{i0}$$

and total final development resources

$$\tilde{DR} = \sum_{i=1}^M \tilde{d}_i$$

the random market return on development assets, $\tilde{r}_D$, is

$$\tilde{DR} = DE_0(1 + \tilde{r}_D)$$

Consequently, under the standard assumptions, the following CAPM-type result should obtain:²⁵

$$E[\tilde{r}_j - r] = \beta_j E[\tilde{r}_D - r]$$

where

$$\beta_j = \frac{Cov(\tilde{r}_D, \tilde{r}_j)}{Var(\tilde{r}_D)}$$

We can confirm this result by assuming that the asset returns, $\tilde{r}_j$, are multivariate normally distributed²⁶ and using the utility functions, U_i :²⁷

$$\max_{a_{ij}} E[U_i(\tilde{d}_i)]$$

leads to the first-order conditions, $\forall i, j$,

$$E[U'_i(\tilde{d}_i)(\tilde{r}_j - r)] = 0$$

Since $Cov(X, Y) = E(XY) - E(X)E(Y)$,

$$Cov(U'_i(\tilde{d}_i), \tilde{r}_j) = -E[U'_i(\tilde{d}_i)]E(\tilde{r}_j - r)$$

and, applying Stein's Lemma,

$$E[U''_i(\tilde{d}_i)]Cov(\tilde{d}_i, \tilde{r}_j) = -E[U'_i(\tilde{d}_i)]E(\tilde{r}_j - r)$$

²⁴ Three things should be noted here. First, r is not the risk-free rate: r is like a risk-free rate because it is a promise made—at least partially—by highly rated donor countries. Second, the position taken by DA i in the quasi risk-free asset is d_{i0} : if d_{i0} is non-negative, then there is no borrowing. (See Footnote 5.) Third, only DAs can take positions in the quasi risk-free asset at r : market segmentation holds.

²⁵ For a critique of the CAPM, see Fama, E. and French, K. (2004). *The Capital Asset Pricing Model: Theory and Evidence*. Journal of Economic Perspectives, 18(3), 25-46.

²⁶ For two examples of CAPM special cases, see Chapter 4 in Huang, C. and Litzenberger, R. (1988). *Foundations for Financial Economics*. Prentice Hall.

²⁷ For discussions of cardinal utility, ordinal utility, utility of consumption, utility over returns, state-dependent utility functions, and state-independent utility functions, see Chapter 1 and Chapter 3 in Ingersoll, J. (1987). *Theory of Financial Decision Making*. Rowman & Littlefield.

Dividing by $-E[U'_i(\tilde{d}_i)]$, defining $\alpha_i = -\frac{E[U''_i(\tilde{d}_i)]}{E[U'_i(\tilde{d}_i)]}$, and summing over i ,

$$\sum_{i=1}^M Cov(\tilde{d}_i, \tilde{r}_j) = \left(\sum_{i=1}^M \frac{1}{\alpha_i} \right) E(\tilde{r}_j - r)$$

Defining $\xi = \left(\sum_{i=1}^M \frac{1}{\alpha_i} \right)$,

$$Cov\left(\sum_{i=1}^M \tilde{d}_i, \tilde{r}_j\right) = \xi E(\tilde{r}_j - r)$$

and

$$E(\tilde{r}_j - r) = \frac{1}{\xi} DE_0 Cov(\tilde{r}_D, \tilde{r}_j)$$

Since

$$E(\tilde{r}_D - r) = \frac{1}{\xi} DE_0 Var(\tilde{r}_D)$$

dividing the two equations produces the CAPM-type result.

It should be noted that DE_0 can be observed. Modeling the random variables is relatively straightforward. The policy function of r , as well as the process through which the rate is set, deserves attention.

Applications

The model of DA behavior proposed in this article recognizes that donations are not limited to concessionality in financing instruments²⁸; for example, the concessionality of GCIs should be taken into account²⁹. Similarly, it recognizes that the concessionality internalized by the beneficiaries should be measured in line with generally accepted pricing methodologies; for instance, because concessionality is the difference between a market—or fair—price and a traded price, there is no need for arbitrary ODA principles, and equity instruments do not pose extraordinary challenges.³⁰ Since commercial assets and development assets are separate—but complementary—asset classes, due to the non-distortionary requirement for NSGOs, pricing and valuation differences contain useful information.

²⁸ See Kolev, N. and Stilwell, J. (2023) *A Sustainable MDB Strategy: The Crucial Importance of Financial Sector Development*. Development Finance Agenda, 8(5), 12-13.

²⁹ So-called “market-based interest rates” are concessional:

<https://treasury.worldbank.org/en/about/unit/treasury/ibrd-financial-products/ibrd-flexible-loan>.

³⁰ See <https://www.oecd.org/en/topics/sub-issues/oda-eligibility-and-conditions/official-development-assistance--definition-and-coverage.html#note-1> and <https://www.oecd.org/en/topics/oda-eligibility-and-conditions.html>.